2-3 Continuity

Learning Targets

- I can identify when/where a function is discontinuous.
- I can classify discontinuities.
- I can write an extended function that removes a removable discontinuity.
- I understand how the Intermediate Value Theorem applies to continuous functions.

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Continuity at a Point

Interior points: A function y = f(x) is continuous at an interior point c if

$$\lim_{x \to c} f(x) = f(c)$$

Endpoints: A function y=f(x) is continuous at an endpoint if

<u>left endpoint</u>

$$\lim_{x \to c^+} f(x) = f(c)$$

<u>right endpoint</u>

$$\lim_{x \to c^-} f(x) = f(c)$$

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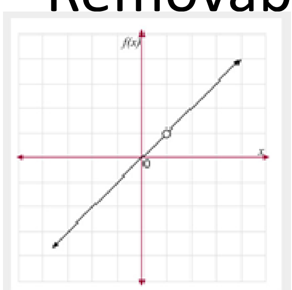
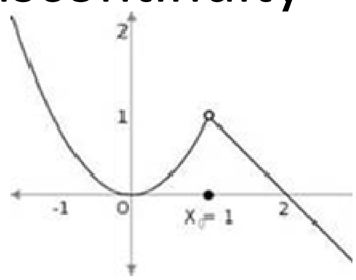
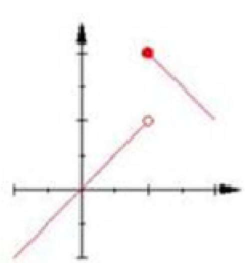
Types of Discontinuity

- Removable Discontinuity





- Jump Discontinuity



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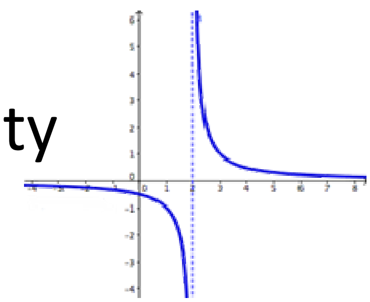
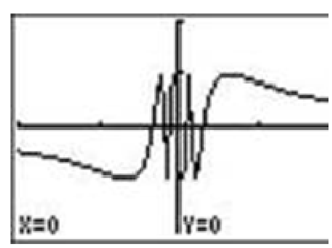
Types of Discontinuity (cont'd)

3. Infinite Discontinuity



4. Oscillating Discontinuity

$y = \sin\left(\frac{1}{x}\right)$



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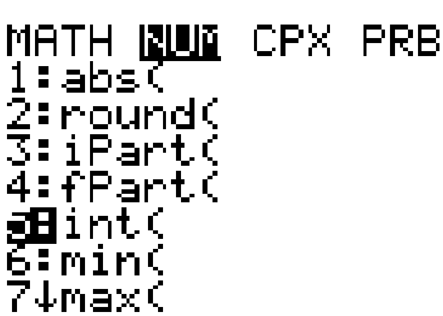
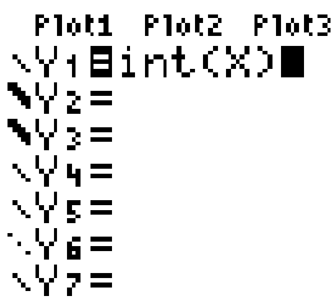
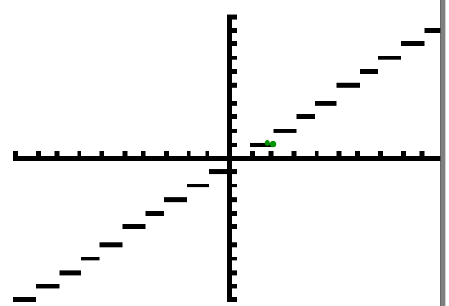
Find the points of continuity and the points of discontinuity for each function.

$$f(x) = [\![x]\!]$$



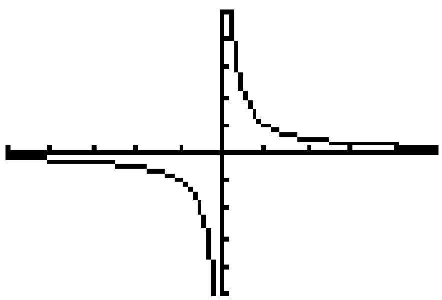




x is discont @ every integer
jump

Find the domain and range for each function and discuss any discontinuities.

$$f(x) = \frac{1}{x}$$



inf discont

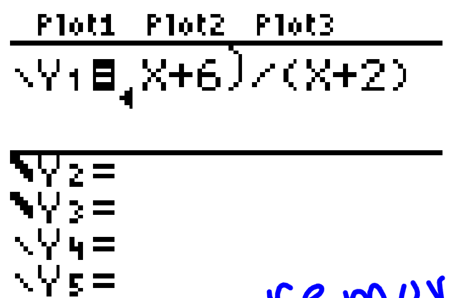
@ $x = 0$

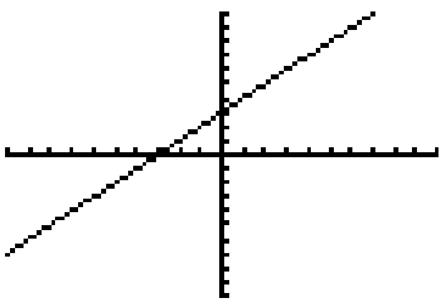
cont

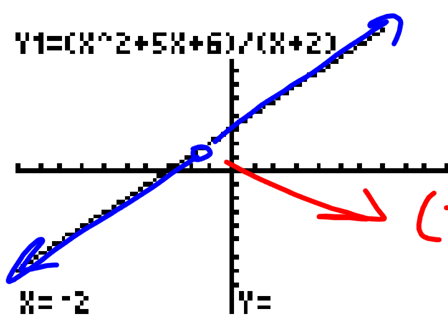
$(-\infty, 0)$ & $(0, \infty)$

$x \neq 0$

$$f(x) = \frac{x^2 + 5x + 6}{x + 2}$$







removable
@ x = 2

(-2, 1)

cont

$(-\infty, -2)$ & $(-2, \infty)$

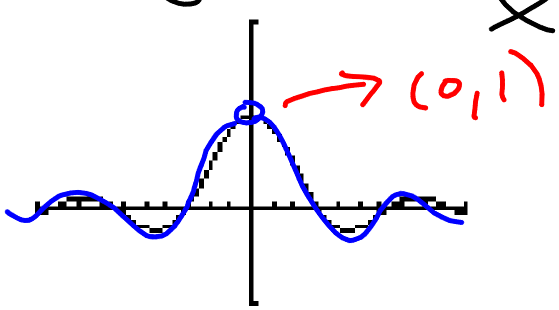
1. $f(x) = \dfrac{x^2 + 7x + 12}{x^2 - 9}$

$f(x) = \dfrac{\cancel{(x+3)}(x+4)}{\cancel{(x+3)}(x-3)} = \dfrac{x+4}{x-3}$

X	Y_1	Y_2
-3	ERROR	-.1667
-2	-.4	-.4
-1	-.75	-.75
0	-1.333	-1.333
1	-2.5	-2.5
2	-6	-6
3	ERROR	ERROR

X= -3

2. $g(x) = \dfrac{\sin x}{x}$



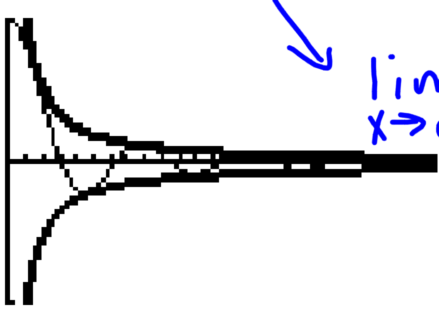
$g(x) = \begin{cases} \dfrac{\sin x}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$

Sandwich Theorem

$$\lim_{x \to \infty} \frac{\sin x}{x} = 0$$

$$-1 \le \sin x \le 1$$

$$-\frac{1}{x} \le \boxed{\frac{\sin x}{x}} \le \frac{1}{x}$$



$$\lim_{x \to \infty} -\frac{1}{x} = 0 \qquad \lim_{x \to \infty} \frac{1}{x} = 0$$

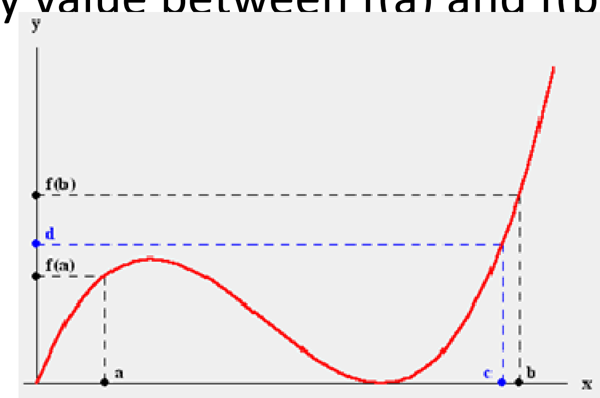
Properties of Continuous Functions

If functions f and g are continuous at x=c, then the following combinations are continuous at x=c.

- Sums: $f+g$
- Differences: $f-g$
- Products: $f \cdot g$
- Constant Multiples: $k \cdot f$ (if k is constant)
- Quotients: $\frac{f}{g}$ (if g(c) does not equal 0)
- Composites: $f \circ g$ and $g \circ f$

Intermediate Value Theorem for Continuous Functions

A function y=f(x) that is continuous on a closed interval [a,b] takes on every value between f(a) and f(b).



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Homework:

p. 84 #1-4, 10-18, 21-25, 41-44

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